Research Paper

Neutrino Oscillation and S_4 Flavor Symmetry

Jong-Chul Park*

Department of Physics, Chungnam National University, Daejeon 34134, Republic of Korea

Received September 17, 2018; accepted September 19, 2018

Abstract

Observations of neutrino oscillations are very strong evidence for the existence of neutrino masses and mixing. From recent experimental results on neutrino oscillation, we find that neutrino mixing angles are quite consistent with the so-called tri-bi-maximal mixing pattern, but the deviation from observational results is non-negligible. However, the tri-bi-maximal mixing pattern is still useful as a leading order approximation and provides a good guideline to search for the flavor symmetry in the neutrino sector. We introduce the S_4 permutation symmetry as a flavor symmetry to the standard model of particle physics with additional particle contents of heavy right-handed neutrinos and scalar fields. Finally, we obtain the tri-bi-maximal mixing pattern as a mixing matrix in the lepton sector within the suggested model. To derive the required unitary mixing matrix for the neutrino sector, the double seesaw mechanism is utilized.

Keywords: Neutrino, Oscillation, Flavor symmetry

I. Introduction

The standard model (SM) of particle physics [1] provides a very successful description of particles and interactions below the electroweak (EW) energy scale $O(100)$ GeV, based on the gauge group $SU(3)_c \times SU_L \times U(1)_Y$ with a chiral representation of fermions. The gauge group of the SM is spontaneously broken to $SU(3)_c \times U(1)_{EM}$ when the Higgs scalar field has a nonzero vacuum expectation value (VEV) on the order of 100 GeV. Through this mechanism, we understand how the elementary particles in the SM have masses. Despite the success of the standard model, the fermion sector introduces major puzzles into the SM. First, within the SM, the neutrinos are massless, but recent observations of neutrino oscillations [2,3] provide strong evidence for the presence of neutrino mass and mixing. Second, the quark sector has the so-called strong CP problem [4].

The observation of the oscillations of solar, atmospheric, accelerator, and reactor neutrinos shows that neutrinos have nonzero masses and flavor mixing similar to quarks [2, 3]. In addition, cosmological observations, such as large scale structure and cosmic microwave background radiation, as well as neutrinoless double β -decay experiments, provide an upper limit for the sum of SM neutrino masses of $O(0.1)$ eV, which is much smaller than the masses of the other SM fermions [2]. Thus, we need to extend the SM to

explain the non-vanishing and very tiny masses of neutrinos. The most popular solution to this problem in the neutrino sector is the so-called seesaw mechanism [5], where one introduces right-handed (RH) neutrinos and suppress the masses of the SM neutrinos via heavy Majorana masses for the RH neutrinos. The original seesaw mechanism can be easily extended to a double seesaw mechanism with additional SM singlets [6]. With the double seesaw mechanism and an appropriate family symmetry, the Dirac-type Yukawa coupling dependence can be removed from the neutrino mass matrix, which is therefore directly proportional to the Majorana mass matrix of heavy RH neutrinos.

We have another question for the neutrino mixing matrix, in addition to the puzzle of the tiny but nonzero neutrino masses. From experimental results of neutrino oscillations, we find that the pattern of observed neutrino mixing angles is quite different from that of the small mixing angles in the quark sector. In the neutrino sector, 2-3 mixing is nearly maximal ($\sin^2\theta_{23} \simeq 1/2$), 1-2 mixing is large ($\sin^2\theta_{12} \simeq 1/3$), but 1-3 mixing is small ($\sin^2\theta_{13} \simeq 0.02$) [2,3]; this is very similar to the so-called tri-bi-maximal mixing pattern [7]. Because of the interesting mixing pattern in the neutrino sector, various flavor symmetries have been suggested to explain the mixing pattern.

In this work, we first review flavor mixing in the lepton sector, neutrino oscillation, and observational results on neutrino masses and mixing angles. Next, by introducing a S_4 flavor symmetry in the lepton and Higgs sectors, we obtain the tri-bi-maximal leptonic mixing matrix.

*Corresponding author
E-mail: jcpark@cnu.ac.kr

II. Flavor mixing and neutrino oscillation

Neutrinos have no masses in the SM because no Yukawa coupling term is allowed for neutrinos. Thus, any unitary transformed states of neutrinos can be chosen as mass eigenstates. Moreover, the unitary mixing matrix for the lepton mass can be absorbed by the unitary transformation of the neutrino mass matrix. If neutrinos have nonzero masses, however, the lepton sector also has a physical mixing matrix, like the quark sector.

The difference between the flavor (or gauge) eigenstates $\nu_\alpha^T \equiv (\nu_e, \nu_\mu, \nu_\tau)$ and the mass eigenstates $\nu_i^T \equiv (\nu_1, \nu_2, \nu_3)$ results in flavor mixing. The mass matrix of charged leptons M_l needs to be diagonalized by a bi-unitary matrix because the mass matrix M_l is neither symmetric nor Hermitian. However, the mass matrix of neutrinos M_ν is symmetric if neutrinos are Majorana particles and can therefore be diagonalized by a single unitary matrix. Thus, if the mass matrices for charged leptons and neutrinos are diagonalized as follows:

$$U_\nu^T M_\nu U_\nu = M_\nu^d, U_l M_l V_l^\dagger = M_l^d, \quad (1)$$

where $M_\nu^d = \text{diag}(m_1, m_2, m_3)$ and $M_l^d = \text{diag}(m_e, m_\mu, m_\tau)$, the physical unitary mixing matrix for neutrinos is given by

$$U_{PMNS} = U_l^\dagger U_\nu, \quad (2)$$

which is known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. For the standard three-neutrino theory, the PMNS matrix is typically parameterized as

$$U_{PMNS} = U_{23}(\theta_{23})U_{13}(\theta_{13}, \delta)U_{12}(\theta_{12}), \quad (3)$$

where U_{ij} is the rotation matrix in the ij plane with a rotation angle θ_{ij} and δ is the Dirac CP phase factor.

Neutrino oscillation is a quantum mechanical phenomenon in which a neutrino created with a specific flavor can be observed as a different flavor later, which is described by the PMNS matrix. The PMNS matrix $U_{\alpha i}$ relates flavor eigenstates ν_α to mass eigenstates ν_i as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \quad (4)$$

The propagation of $|\nu_i\rangle$ can be described by a plane wave solution because $|\nu_i\rangle$ are mass eigenstates. Thus, the transition amplitude that a neutrino of flavor α will later be measured as a flavor β is given by

$$\langle \nu_\beta | \nu_\alpha \rangle = \sum_j U_{\beta j}^* U_{\alpha j} e^{-i(E_j t - \vec{p}_j \cdot \vec{x})}, \quad (5)$$

where E_j and $\vec{p}_j$ are the energy and the three-dimensional momentum of the mass eigenstate i ; t and $\vec{x}$ are the relative time and position from the starting time and position of the propagation, respectively. Recasting p_j and t in terms of $E_j = E, m_j$, and $x = L$ for the fixed neutrino energy, the oscillation probability of a neutrino of flavor α is

$$P_{\alpha \rightarrow \beta} = \delta_{\alpha\beta} - 4 \sum_{i < j} \text{Re}(U_{\beta i}^* U_{\alpha j} U_{\beta i} U_{\alpha j}^*) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right) + 2 \sum_{i < j} \text{Im}(U_{\beta i}^* U_{\alpha j} U_{\beta i} U_{\alpha j}^*) \sin\left(\frac{\Delta m_{ij}^2 L}{2E}\right), \quad (6)$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$

III. Observational results

As can be seen from Eq. (6), observational results for neutrino oscillation depend on the following parameters: the mass-squared differences Δm_{ij}^2 , mixing angles θ_{ij} , and the Dirac CP phase δ . To obtain oscillation parameters, a global fit analysis of all experimental data should be performed because each observable is determined by a combination of several parameters: (i) The mixing angle θ_{12} (the so-called solar angle θ_{sol}) is obtained from solar neutrino experiments combined with Kamland, accelerator, and reactor neutrino data. (ii) The atmospheric mixing angle θ_{23} (θ_{atm}) is determined by atmospheric neutrino experiments together with accelerator neutrino experiments. (iii) The last mixing angle θ_{13} is obtained from a combination of several reactor neutrino experiment results.

The observed values of the oscillation parameters in a vacuum based on the global fit are given by PDG 2018 [2] as follows:

$$\sin^2(\theta_{12}) = 0.307 \pm 0.013 \text{ with } \Delta m_{21}^2 (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2,$$

$$\sin^2(\theta_{23}) = 0.417_{-0.028}^{+0.025} \text{ or } 0.597_{-0.030}^{+0.024} \text{ with } \Delta m_{32}^2 = (2.51 \pm 0.05) \times 10^{-3} \text{ eV}^2,$$

$$\sin^2(\theta_{13}) = (2.12 \pm 0.08) \times 10^{-2}. \quad (7)$$

The sign of Δm_{32}^2 determines the mass hierarchy in the SM neutrino sector: either normal mass hierarchy with $\Delta m_{32}^2 > 0$ or inverted mass hierarchy with $\Delta m_{32}^2 < 0$. For the above results in Eq. (7), the normal mass hierarchy is assumed. Note that neutrino oscillation should be modified in matter as a result of a particle physics process known as the Mikheyev-Smirnov-Wolfenstein effect [8,9].

Comparing the neutrino mixing matrix with the observational data, one can find one interesting feature in the neutrino mixing pattern. The neutrino mixing angles from the global fit analysis are given by

$$\theta_{12} = 33.6^\circ, \theta_{23} = 40.2^\circ \text{ or } 50.6^\circ, \theta_{13} = 8.37^\circ, \quad (8)$$

which are quite close to the following values:

$$\frac{1}{\sqrt{3}} \simeq \sin 35.3^\circ, \frac{1}{\sqrt{2}} = \sin 45^\circ, 0 = \sin 0^\circ. \quad (9)$$

Thus, one can use a simple and interesting form for the neutrino mixing matrix, the so-called tri-bi-maximal (TBM) mixing matrix, which is

$$U_{\text{TBM}} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix}. \quad (10)$$

After the discovery of this pattern, many researchers have investigated the origin of this interesting pattern. Currently, the discrepancy between the TBM mixing pattern and the experimental values is greater than 2σ as a result of much improved experimental observations. However, the TBM mixing pattern is still a very useful leading order approximation to find a discrete flavor symmetry behind the neutrino mixing. Moreover, the discrepancy may originate from the small violation of the flavor symmetry.

IV. Tri-bi-maximal mixing from s_4 flavor symmetry

The S_4 discrete symmetry is the permutation symmetry of four objects, $\{1234\} \rightarrow \{ijkl\}$, where $\{ijkl\}$ is a permutation of $\{1234\}$. S_4 has 24 elements and five irreducible representations of $\mathbf{3}, \mathbf{3}', \mathbf{2}, \mathbf{1}$ and $\mathbf{1}'$ with the multiplication rules

$$\begin{aligned} \mathbf{3} \times \mathbf{3} &= \mathbf{1}(11 + 22 + 33) + \mathbf{2}(11 + \omega^2 22 + \omega 33, 11 + \omega 22 + \omega^2 33) \\ &\quad + \mathbf{3}_s(23 + 32, 31 + 13, 12 + 21) + \mathbf{3}_a(23 - 32, 31 - 13, 12 - 21), \\ \mathbf{3}' \times \mathbf{3}' &= \mathbf{1} + \mathbf{2} + \mathbf{3}_s + \mathbf{3}_a', \mathbf{3} \times \mathbf{3}' = \mathbf{1}' + \mathbf{2}' + \mathbf{3}_s' + \mathbf{3}_a, \\ \mathbf{2} \times \mathbf{2} &= \mathbf{1}(12 + 21) + \mathbf{1}'(12 + 21) + \mathbf{2}(22, 11), \\ \mathbf{1} \times \mathbf{1}(\mathbf{1}') &= \mathbf{1}(\mathbf{1}'), \mathbf{1} \times \mathbf{1}' = \mathbf{1}, \end{aligned} \quad (11)$$

where $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, i.e., a cubic root of unity. One of the interesting features of S_4 is that it has both S_3 and A_4 ; thus, both couplings of $\mathbf{3} \times \mathbf{3} \times \mathbf{3} \rightarrow \mathbf{1}$ and $\mathbf{2} \times \mathbf{2} \times \mathbf{2} \rightarrow \mathbf{1}$ are allowed, as can be seen from Eq. (11).

Under the S_4 flavor symmetry, the particle contents of our model are assigned as in Table I. For the particle assignment, $f_i f_j' : \mathbf{3} \times \mathbf{3} = \mathbf{3} + \mathbf{3}' + \mathbf{2} + \mathbf{1}$ with $f \equiv l^c$ and $f' \equiv L$. Thus, we can find $S_4 \times Z_2$ invariant Yukawa couplings,

$$\lambda_1 f_i f_j' \Phi_{Hk} : \mathbf{3} \times \mathbf{3} \rightarrow \mathbf{1} = \lambda_1 [(f_2 f_3' + f_3 f_2') \Phi_{H1} + (f_3 f_1' + f_1 f_3') \Phi_{H2} + (f_1 f_2' + f_2 f_1') \Phi_{H3}], \quad (12)$$

$$\lambda_2 f_i f_j' \Phi_{Hk} : \mathbf{3}' \times \mathbf{3}' \rightarrow \mathbf{1} = \lambda_2 [(f_2 f_3' - f_3 f_2') \chi_{H1} + (f_3 f_1' - f_1 f_3') \chi_{H2} + (f_1 f_2' - f_2 f_1') \chi_{H3}]. \quad (13)$$

By choosing the VEVs for the scalar fields as $\langle \Phi_{Hi} \rangle = \langle \Phi \rangle$ and $\langle \chi_{Hi} \rangle = \langle \chi \rangle$, we can obtain the mass matrix for charged leptons,

$$M_l = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix}, \quad (14)$$

where $a=0$, $b = \lambda_1 \langle \Phi \rangle + \lambda_2 \langle \chi \rangle$, and $c = \lambda_1 \langle \Phi \rangle + \lambda_2 \langle \chi \rangle$. Therefore, the diagonalizing matrix U_l for $M_l^2 = M_l^\dagger M_l$ is given by a tri-maximal mixing form,

$$U_l = \begin{pmatrix} 1/\sqrt{3} & \omega/\sqrt{3} & \omega^2/\sqrt{3} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & \omega^2/\sqrt{3} & \omega/\sqrt{3} \end{pmatrix}. \quad (15)$$

Next, let us focus on the neutrino sector. With the particle assignment in Table I, the neutrino mass matrix M_ν is proportional only to the heavy neutrino mass matrix M^{SS} because of the double seesaw mechanism [6]. Thus, we only need to examine the mass matrix M^{SS} to obtain the mixing matrix diagonalizing M_ν . For heavy neutrinos $S_{1,2,3}$, $S_i S_j : (\mathbf{1} + \mathbf{2}^-) \times (\mathbf{1} + \mathbf{2}^-) = \mathbf{2}_1^- + \mathbf{2}_2^- + \mathbf{2}_3 + \mathbf{1}_1 + \mathbf{1}_2 + \mathbf{1}'$ under the $S_4 \times Z_2$ symmetry. Thus, $S_4 \times Z_2$ invariant Yukawa couplings are given by

$$\begin{aligned} S_i S_j \phi_{Sk} : \mathbf{1}_1(S_i S_j) \times \mathbf{1}(\phi_{Sk}) &\rightarrow \mathbf{1} = \lambda_1' S_2 S_2 \phi_{S1}, \\ \mathbf{1}_2(S_i S_j) \times \mathbf{1}(\phi_{Sk}) &\rightarrow \mathbf{1} = \lambda_2' (S_1 S_3 + S_3 S_1) \phi_{S1}, \\ \mathbf{2}_3(S_i S_j) \times \mathbf{2}(\phi_{Sk}) &\rightarrow \mathbf{1} = \lambda_3' (S_3 S_3 \phi_{S3} + S_3 S_3 \phi_{S2}). \end{aligned} \quad (16)$$

Taking the vacuum direction as $\langle \Phi_{S2} \rangle = \langle \Phi_{S3} \rangle = \langle \phi \rangle$, the mass matrix for heavy neutrinos is obtained as

$$M^{\text{SS}} = \begin{pmatrix} a' & 0 & b' \\ 0 & c' & 0 \\ b' & 0 & a' \end{pmatrix}, \quad (17)$$

where $a' = \lambda_3' \langle \phi \rangle$, $b' = \lambda_2' \langle \phi_{S1} \rangle$, and $c' = \lambda_1 \langle \phi_{S1} \rangle$. M^{SS} is diagonalized by a bi-maximal mixing matrix U_s , which is given by

Table I. Particle contents and S_4 assignments. $S_{1,2,3}$ are SM singlet heavy neutrinos, $\Phi_H = (\Phi^0, \Phi^-)^T$ and $\chi_H = (\chi^0, \chi^-)^T$ are Higgs doublets, and ϕ_s are singlet scalars. Superscript “ $-$ ” of $S_{1,3} \sim \mathbf{2}^{(-)}$ stands for odd under an additional Z_2 parity.

Fields	L	l^c	S	Φ_H	χ_H	ϕ_s	VEVs
S_4	$\mathbf{3}$	$\mathbf{3}$	$S_2 \sim \mathbf{1},$ $S_{1,3} \sim \mathbf{2}^{(-)}$	$\mathbf{3}$	$\mathbf{3}'$	$\phi_{S1} \sim \mathbf{1},$ $\phi_{S2,3} \sim \mathbf{2}$	$\langle \Phi_{Hi} \rangle = \langle \Phi \rangle, \langle \chi_{Hi} \rangle = \langle \chi \rangle,$ $\langle \Phi_{S1} \rangle, \langle \Phi_{S2} \rangle = \langle \Phi_{S3} \rangle = \langle \phi \rangle$

$$U_S = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}, \quad (18)$$

because M_ν is proportional to $M^{(SS)}$, the diagonalizing matrix U_ν for M_ν satisfies $U_\nu = U_S$.

The physical unitary mixing matrix for neutrinos is given by $U_{PMNS} = U_l^\dagger U_\nu$, as shown in Eq. (2). From Eq. (15) and Eq. (18) with the relation $U_\nu = U_S$, we finally obtain the TBM mixing matrix for neutrinos $U_{PMNS} = U_l^\dagger U_\nu = U_l^\dagger U_S = U_{TBM}$. Here, we take ν_3 as a new mass eigenstate by a field redefinition.

V. Conclusions

Recent experimental results for neutrino oscillations show that mixing angles in the neutrino sector are quite close to the so-called tri-bi-maximal mixing pattern, although the difference between the experimental observation and the TBM mixing pattern is larger than 2σ as a result of the good sensitivities of improved experiments. Nevertheless, the TBM mixing pattern is still a useful guideline to seek a symmetry hidden in the neutrino mixing. In this paper, we used the S_4 discrete

symmetry as a flavor symmetry and introduced three heavy neutrinos and additional scalar fields. With the particle assignment under S_4 symmetry, we obtained the TBM mixing matrix for neutrinos. The deviation from the exact TBM mixing matrix may be explained by the effect of the slight violation of a flavor symmetry in the neutrino sector.

Acknowledgements

This work was supported by the research fund of Chungnam National University.

Reference

- [1] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
- [2] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D 98, 030001(2018).
- [3] J. K. Ahn *et al.* [RENO Collaboration], Phys. Rev. Lett. 108, 191802 (2012).
- [4] J. E. Kim and G. Carosi, Rev. Mod. Phys. 82, 557 (2010).
- [5] P. Minkowski, Phys. Lett. B 67, 421 (1977).
- [6] J. E. Kim and J. C. Park, JHEP 0605, 017 (2006).
- [7] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B 530, 167 (2002).
- [8] L. Wolfenstein, Phys. Rev. D 17, 2369 (1978).
- [9] S. P. Mikheev and A. Y. Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985).