Research Paper

Analysis of Impedance of Microwave Parallel Stripline Resonator Discharge Source for Application of Microplasma System at Atmospheric Pressure

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Abstract

The microplasma system that uses a stripline resonator is a promising plasma system because, capitalizing on the resonance property of the plasma source, it can readily generate plasma under atmospheric conditions without the assistance of an additional matching network. However, precise determination of the resonance frequency before device manufacturing is very important in practice. Therefore, the evolution trends of the resonance frequency of the discharge have to be investigated, based on source impedance analysis, to get an insight into the discharge matching condition. In this paper, a means of determining the discharge source impedance, called conformal mapping, and its application to the microwave parallel stripline resonator discharge source is presented and discussed.

Keywords: Atmospheric plasma, Source impedance, Conformal mapping

I. Introduction

In many rf systems, some of the output power of the rf generator does not reach the load because of power loss in the line and matcher. When a plasma chamber is used as the load, the power loss in the chamber is non-intuitive because plasma is a non-linear dielectric material. Thus, to accurately measure the power loss in the chamber, an rf sensor can be connected to the chamber. Microplasmas have recently received much attention for development and application. For a combination of the potentials of low temperature plasmas, which have the advantage of being in the micro scale, the discharge creates a highly reactive environment that contains charged particles, excited species, radicals, and photons; this is applicable to various fields, including bio-medical applications (treatment of living tissues, tissue sterilization, and blood coagulation), dental treatment, displays, radiation sources, micro-chemical analysis systems, gas analyzers, and photo-detectors. Microplasmas can be generated over wide pressure range from a few mTorr up to a few atmospheres. Typically, operation of microplasma systems

at atmospheric pressure is more favored as its size can be reduced by eliminating the micro-pump and it only requires low power, making its integration into microsystems and portable devices possible [1].

Recently, the microwave parallel stripline resonator discharge source has been widely investigated due to its ability to use the source resonance to generate plasma even under difficult conditions such as that at atmospheric pressure, without assistance from an additional matching box. However, precise determination of the resonance frequency before device manufacturing is very important in practice. Therefore the evolution trends of the resonance frequency of the discharge have to be investigated, based on source impedance analysis, to get an insight into the discharge matching condition. In this paper, a means of determining the discharge source impedance, called conformal mapping, and its application to the microwave parallel stripline resonator (MPSR) discharge source is presented and discussed.

II. Theory

II-1. Motive

In this section, we the first explain our motivation for this work before we beginning calculations. Figure 1

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Figure 1. (Color online) The reflection coefficient (S11) of the MPSR calculated using the CST microwave studio software.

is an example of the S11 parameter of the MPSR, obtained using commercial CST microwave studio software (CST-Computer Simulation Technology Company). As shown in Fig. 1, the parameter spectrum has a sharp minimum value at approximately 0.74 MHz, reflecting strong resonance in the device. A simple calculation based on Fig. 1 shows that the 3 dB quality factor ($Q = \frac{f_{Y \min}}{\Delta f_{Y \min - 3dB}}$) of the MPSR source is very high at 1747. This indicates the ability of the device for high efficiency discharge operations under resonance conditions. Because of the high quality factor of the device, the precise determination of the resonance frequency before device manufacturing is very important. Therefore, the evolution trends of the resonance frequency with change in other input parameters such as the location of the feeding point of the input power, the characteristic stripline impedance, and the impedance of the discharge gap should be investigated to get an insight into the discharge matching condition.

One can apply an electric circuit model to estimate the resonant frequency and net impedance of an MPSR and thereby determine the key parameters that control the performance of the device. This results in the formulation of closed-form expressions that are useful for designing an MPSR and analyzing the designs. However, MPSR geometry is not like that of a stripline or other well-known geometries. Therefore, the device parameters must be determined carefully. There are various methods to do this, such as finite difference methods, Green's functions, and conformal mapping. In this paper, the characteristic impedance, capacitance,

and inductance of the design are calculated using the conformal mapping method. The data obtained after calculation, simulation with a commercial program, and conducting an experiment based on an MPSR device are presented for comparison.

II-2. Conformal mapping

Conformal mapping is an important means of solving a wide range of physical problems, such as those in fluid flow, aerodynamics, thermomechanics, electrostatics, and elasticity [2-4], because it helps to reduce complicated mathematical problems in complex geometry and 2D symmetry to simpler ones. Particularly, in electrostatics and transmission lines, conformal mapping has been used to estimate the potential, electric field, and capacitance [5-10]. The difference between conformal mapping and normal mapping is that the mapping function used in conformal mapping is the analytic and nonzero version of its derivative on the mapped region. Therefore, these transformations have following properties.

First, a harmonic function satisfying the Laplace's equation is transformed into a harmonic function.

Second, with conformal mapping, the Dirichlet and Neumann boundary conditions remain unchanged in the transformed region. Third, the conformal function preserves the capacitances of the corresponding conductors. The second property of conformal mapping implies that, instead of solving a boundary value problem directly on the original plane, it can be transformed into a simpler problem on the mapped plane. The third property is the key to estimating capacitances where a two-dimensional region with an abnormal distribution of conductors or boundaries is transformed into a region where capacitance is known, for example, in parallel plates. A widely used conformal transformation for capacitance calculation is the Schwarz-Christoffel mapping [2,11].

II-3. Schwarz-Christoffel transformation

The Schwartz-Christoffel transformation transforms the real axis of the original map (on the z -plane) into a polygonal curve (on the w -plane). It also transforms the upper half-plane of the real axis into a domain bounded by this polygonal curve, as shown in Fig. 2 [2,12]. The transformation is given by:

$$\frac{dw}{dz} = A(z-x_1)^{\alpha_1/\pi-1}(z-x_2)^{\alpha_2/\pi-1} \dots (z-x_n)^{\frac{\alpha_n}{\pi}-1} \quad (1)$$

Figure 2. (Color online) Schwarz-Christoffel conformal mapping for a half-plane.

Or

$$w = A \int (z-x_1)^{\alpha_1/\pi-1} (z-x_2)^{\alpha_2/\pi-1} \dots (z-x_n)^{\alpha_n/\pi-1} + B \quad (2)$$

where A and B are complex constants that determine the size, orientation, and position of the polygon. $(x_1, x_2, \dots, x_n)$ are the points on the real axis corresponding to the polygon vertices and $(\alpha_1, \alpha_2, \dots, \alpha_n)$ are the interior angles. If a point on the real axis of the z -plane is at infinity, for instance x_n , then Eq. (1) becomes

$$\begin{aligned} \frac{dw}{dz} &= x_n A (z-x_1)^{\frac{\alpha_1}{\pi}-1} (z-x_2)^{\frac{\alpha_2}{\pi}-1} \dots (z-x_{n-1})^{\frac{\alpha_{n-1}}{\pi}-1} \\ &= A' (z-x_1)^{\frac{\alpha_1}{\pi}-1} (z-x_2)^{\frac{\alpha_2}{\pi}-1} \dots (z-x_{n-1})^{\frac{\alpha_{n-1}}{\pi}-1} \end{aligned} \quad (3)$$

Eq. (3) shows that the factor involving x_n is absent. This condition is often used in case where the point at infinity on the plane maps to one of the vertices of the plane polygon. Another note significant for application is that infinite open polygons can be considered as the limits of closed polygons.

Previous studies have shown that the TEM wave mode is favored on the stripline and that the electric field along a transmission line satisfies the Laplace's equation [1]. This implies that the electric field around a stripline has the properties of an electrostatic field. Therefore, conformal mapping can be used to determine the resonator capacitance.

III. Calculation result and discussion

III-1. Capacitance calculation

Figure 3 shows the sample of electric field vectors distribution along two branches of the resonator from a side view (a), and front view (b). The tangential com-

Figure 3. (Color online) Example of electrical field distributions along the two legs of the resonator in (a) side view and (b) front view; (c) the corresponding capacitance network for a resonator operated in the odd resonant mode.

ponent of the electric field is zero at the symmetry plane ff' , called virtue ground. This is equivalent to placing an electric wall at ff' ; in other words, the potential will be zero at the electric boundary. Therefore, each impedance that connects between two branches of the resonator can be considered as two halves connected to the virtual ground. Figure 3(c) shows the equivalent capacitance network for the resonator operated in an odd resonant mode. In this circuit, C_s is capacitance of strip to ground of one branch, C_m is the mutual capacitance between two branches, and C_g is the capacitance at discharge gap. The mutual and discharge gap capacitance impedance are separated in two halves connect to the virtual ground, corresponding to a double value for each half as shown in Fig. 3(c). We will use the conformal mapping to determine the strip to ground capacitance, mutual capacitance, and discharge gap capacitance of the resonator. These capacitances will in turn help deduce the characteristic impedance of the resonator, effective electric constant, and resonance frequency.

III-2. Mutual capacitance

The sequence for calculating mutual capacitance between two legs of the resonator in a rectangle is shown in Fig. 4 using the (a) original structure, (b) intermediate structure, upper half-plane, (c) intermediate structure with scaled axis, and (d) parallel plane capacitor. First, we use the Schwarz-Christoffel transformation, given in Eq. (2), to calculate the capacitance caused by the field inside two strips without fringe field, which is due to edge effect, assuming a wide strip width.

Transformation from z -plane to t -plane

The mapping equation from the z -plane into the t -plane gives as follows:

Figure 4. (Color online) Mapping sequential for calculating C_m , including inside gap capacitance, (a) original structure, (b) intermediate structure, upper half-plane, (c) intermediate structure with normalized axis; and (d) parallel plane capacitor.

$$z = A \int_0^t (t-t_2)^{\alpha_2/\pi-1} (t-t_3)^{\alpha_3/\pi-1} (t-t_7)^{\alpha_7/\pi-1} (t-t_8)^{\alpha_8/\pi-1} dt + B \quad (4)$$

Because the angle at points z_4 , z_5 , and z_6 equal π , and they have unit value, they are not presented in Eq. (4). Two points z_1 and z_9 were set infinity; thus, they are also not presented in Eq. (4). The angles of the boundary in the z -plane are $\alpha_2 = \alpha_3 = \alpha_7 = \alpha_8 = \pi/2$, and the corresponding points in the t -plane are $t_7 = -t_3 = 1$ and $t_8 = -t_2$ due to symmetry. Hence, Eq. (4) can be manipulated to form the following:

$$z = A' \int_0^t [(t-k^2t^2)(t-t^2)]^{-1/2} dt + B \quad (5)$$

$$z = A' F(t, k) + B$$

where $k = \frac{1}{t_8}$, and

$$F(t, k) = \int_0^t [(t-k^2t^2)(t-t^2)]^{-\frac{1}{2}} dt \quad (6)$$

$F(t, k)$ is the incomplete elliptic integral of the first type, k is its modulus [4,13], when $t = 1$; it is a complete elliptic integral and is denoted by the symbol $K(k) = F(1, k)$. These elliptic integrals exhibit the following properties:

$$F(\pm 1/k, k) = \pm K(k) + iK(k') \quad (7)$$

where k' is the complementary modulus and $k' = \sqrt{1-k^2}$. The inverse function of $F(t, k)$, that is pressed as functions of z and k , is as follows:

$$t = F^{-1}(z, k) = sn(z, k) \quad (8)$$

Constants A' and B can be obtained from the boundary conditions as follows:

$$t_7 = 1 \leftrightarrow z_7 = \frac{h}{2} \quad (9a)$$

$$t_5 = 0 \leftrightarrow z_5 = 0 \quad (9b)$$

substituting the boundary conditions Eqs. (9a) and (9b) into Eq. (5), we obtain

$$\frac{h}{2} = A' F(1, k) + B = A' K(k) + B \quad (10a)$$

$$0 = A' F(0, k) + B = A' 0 + B \quad (10b)$$

Solving the equations in system 10, we obtained $B = 0$ and $A' = \frac{h}{2K(k)}$. Therefore, from Eq. (5), the mapping function becomes

$$z = \frac{h}{2K(k)} F(t, k), \quad (11)$$

and the position of t_6 on the t -plane can be determined according to its boundary condition as follows:

$$z_6 = \frac{h}{2K(k)} F(t_6, k) = \frac{g}{2} \quad (12a)$$

$$\Rightarrow t_6 = sn\left(K(k) \frac{g}{h}, k\right) \quad (12b)$$

To obtain t_6 and t_8 , the modulus k is required. We can use the following approximation [12]

$$\frac{K(k)}{K(k')} = \begin{cases} \frac{1}{\pi} \ln \left(\frac{2(1+\sqrt{k})}{(1-\sqrt{k})} \right), & 0.5 \leq k^2 \leq 1 \\ \ln \left(\frac{2(1+\sqrt{k'})}{1-\sqrt{k'}} \right), & 0 < k^2 < 0.5 \end{cases}$$

and the solution is

$$\begin{cases} k' = \left(\frac{e^{\pi m} - 2}{e^{\pi m} + 2} \right)^2, & m \leq 1 \\ k = \left(\frac{e^{\pi m} - 2}{e^{\pi m} + 2} \right)^2, & m > 1 \end{cases}$$

With $m = \frac{K(k)}{K(k')}$, the boundary conditions at z or z can be obtained as follows:

$$\begin{aligned} z_2 &= \frac{h}{2K(k)} F(t_2, k) \\ &= \frac{h}{2K(k)} F(-1/k, k) \\ &= \frac{h}{2K(k)} [-K(k) + jK(k')] = -\frac{h}{2} + jl \\ \Rightarrow \frac{K(k)}{K(k')} &= \frac{h}{2l} \end{aligned} \quad (13)$$

In our work $m = \frac{h}{2l} < 1$.

Transformation from t -plane to t' -plane

The transformation from the upper-half plane t -plane to the upper-half plane t' -plane only scales the real axis to $t_6 = 1$, with the scaling coefficient $1/t_6 = 1$ as follows

$$t_6' = -t_4' = 1, \quad (14)$$

$$t_7' = -t_3' = \frac{t_7}{t_6} = \frac{1}{t_6}, \quad (15)$$

$$t_8' = -t_2' = \frac{t_8}{t_6} = \frac{1}{kt_6} \quad (16)$$

Transformation from t' -plane to w -plane

Now, Eq. (2) is used to map from the t' -plane to the w -plane:

$$\begin{aligned} w &= A_1 \int_0^{t'} (t' - t_2')^{-1/2} (t' - t_4')^{-1/2} (t' - t_6')^{-1/2} \\ &\quad (t' - t_8')^{-1/2} dt + B_1 \end{aligned} \quad (17)$$

By substituting conditions Eq. (14) into Eq. (17), and simplifying to an elliptic integral, we have

$$w = A_1' F(t', k_1) + B_1, \quad (18)$$

where $k_1 = \frac{1}{t_8'} = ksn(K(k) \frac{q}{h}, k)$. Similarly, as the first step, constants A_1' and B can be obtained from the boundary conditions. The solutions are $A_1' = \frac{h}{2K(k_1)}$, $B_1 = 0$. Therefore, the transformation function for this step is

$$w = \frac{h}{2K(k_1)} F(t', k_1). \quad (19)$$

By substituting the boundary conditions of t' -plane into Eq. (19), the position of points in w -plane corresponding to those in t' -plane are

$$\begin{aligned} w_2 &= \frac{h}{2K(k_1)} F(-1/k_1, k_1) \\ &= \frac{h}{2K(k_1)} [-K(k_1) + jK(k_1')] \\ &= -\frac{h}{2} + j\frac{h}{2} K(k_1') \end{aligned} \quad (20)$$

$$\begin{aligned} w_3 &= \frac{h}{2K(k_1)} F(t_3', k_1) \\ &= \frac{h}{2K(k_1)} F\left(\frac{1}{-t_y}, k_1\right) = u_3 + jv_3 \end{aligned} \quad (21)$$

$$w_4 = \frac{h}{2K(k_1)} F(-1, k_1) = -\frac{h}{2} \quad (22)$$

where $k_1' = \sqrt{1 - k_1^2}$. Therefore, the mutual capacitance per unit length caused by the two legs of the resonator (without the edge effect) is

$$C_m = \epsilon_0 \epsilon \frac{K(k_1') - v_3}{2K(k_1)} \quad (23)$$

The half-side capacitance C_1 per unit length of the discharge gap (without the edge effect) is given as

$$C_m = \epsilon_0 \epsilon \frac{v_3}{2K(k_1)} \quad (24)$$

However, as mentioned above, the strip width of the resonator is not infinitely wide; therefore, the contribution of the fringe field in the plane normal to the strip line toward the capacitance must be considered. To estimate the edge effect in this case, we consider the capacitance per unit length between two

Figure 5. (Color online) Mapping sequential for calculating the capacitance between two strips, (a) original structure, (b) intermediate structure, upper half-plane, (c) intermediate structure with normalized axis, and (d) parallel plane capacitor.

finite parallel planes as shown in Fig. 5. A finite parallel plane capacitor shown in Fig. 5(a) with fringe field at two ends can be transformed to an infinite parallel plane capacitor with uniform electric field by applying conformal mapping method. Therefore, the capacitance obtained from the geometry in Fig. 5(d) includes the contribution of the edge effect of the parallel capacitor, as shown in Fig. 5(a). The contribution of the fringe field to the capacitance for this case can be determined from the different values of the capacitances of the infinite parallel capacitor and finite parallel capacitor.

Similarly, the capacitance of parallel capacitors is given as

$$C_{pe} = C_m = \epsilon_0 \epsilon \frac{K(k_2')}{2K(k_2)} \quad (25)$$

where $k_2 = \exp\left(\frac{-\pi W}{h}\right)$ and $k_2' = \sqrt{1 - k_2^2}$. The capacitance per unit length determined from the structure of Fig. 5(a) is

$$C_{pl} = \epsilon_0 \epsilon \frac{W}{h}. \quad (26)$$

We can determine the contribution of the capacitance per unit length of the stripline direction caused by the fringe field, as shown in Fig. 6, as

$$\Delta C_{edge} = \epsilon_0 \epsilon \left[\frac{K(k_2')}{2K(k_2)} - \frac{W}{h} \right] \quad (27)$$

Figure 6. (Color online) Edge effect contribution.

Figure 7. (Color online) Magnified view of the discharge gap with a simple capacitance model for calculation.

Applying Eq. (27) to Eq. (23), we have

$$C_m = \epsilon_0 \epsilon \left(W \frac{(K(k_1') - v_3)}{2K(k_1)} + l \left[\frac{K(k_2')}{2K(k_2)} - \frac{W}{h} \right] \right) \quad (28)$$

and the half-side capacitance C_1 including the effect of discharge gap is given as

$$C_1 = \epsilon_0 \epsilon_1 \left(W \frac{v_3}{2K(k_1)} + v_3 \left[\frac{K(k_2')}{2K(k_2)} - \frac{W}{h} \right] \right) \quad (29)$$

III-3. Discharge gap capacitance

The discharge gap capacitance of the resonance can be given as

$$C_g = C_1 + C_2 + C_3 \quad (30)$$

where C_1 , C_2 , and C_3 are the capacitances of the left-half (inside the dielectric with dielectric constant ϵ_1), right-half (in air or plasma), and across the discharge gap, respectively; g is the discharge gap width; t is the strip thickness; h is the distance between two parallel legs of the resonator as shown in Fig. 7; and W is the gap length (see Fig. 1). The cross capacitance C_3 can be obtained as

$$C_3 = \frac{\epsilon_0 \epsilon_2 t W}{g} \quad (31)$$

where ϵ_2 is 1 in air, and ϵ_{peff} is the effective plasma permittivity in the plasma region. Here, ϵ_{peff} is dependent on plasma parameters (pressure, density,

Figure 8. (Color online) Mapping sequential for calculating C_2 with (a) original structure and (b) parallel plane capacitor.

etc.) and also on the plasma size in the discharge gap.

The left-half component C_1 was calculated in the last section. The sequential schematic for calculating C_3 by applying Schwarz-Christoffel transformation is shown in Fig. 8.

The sequence for calculating C_2 from two coplanar strips is also shown in Fig. 8. Applying the Schwarz-Christoffel transformation to map the upper half plane of z -plane to a rectangle in the w -plane and simplifying the expression similar to the above section, we obtained the mapping function of this transformation as

$$w = F(z, k_3) \quad (32)$$

where k_3 is g/h , and its compensation, k_3' , is $\sqrt{1 - k_3^2}$. The capacitance of the air side of the gap (without the fringe effect) is

$$C_2 = \epsilon_0 (W + 2t) \frac{K(k_3')}{2K(k_3)}. \quad (33)$$

The fringe effect (Fig. 9) is

$$C_2 = \epsilon_0 \epsilon_1 \left((W + 2t) \frac{K(k_3')}{2K(k_3)} + K(k_3') \left[\frac{K(k_4')}{2K(k_4)} - \frac{W}{2K(k_3)} \right] \right) \quad (34)$$

where $k_4 = \exp\left(-\frac{\pi W}{2K(k_3)}\right)$, and $k_4' = \sqrt{1 - k_4^2}$. From Eqs.

(29), (30), (31), and (34), the discharge gap capacitance including the edge effect is

$$C_2 = \epsilon_0 \epsilon_1 \left(W \frac{v_3}{2K(k_1)} + v_3 \left[\frac{K(k_2')}{2K(k_2)} - \frac{W}{h} \right] \right) + \epsilon_0 \epsilon_2 \left((W + 2t) \frac{K(k_3')}{K(k_3)} + K(k_3') \left[\frac{K(k_4')}{2K(k_4)} - \frac{W}{2K(k_3)} \right] \right) + \frac{\epsilon_0 \epsilon_2 t W}{g} \quad (35)$$

Figure 9. (Color online) Gap capacitance with the edges.

Figure 10. (Color online) Schematic of strip to ground capacitance of (a) the original structure and (b) a half-strip to ground capacitance that is mapped onto (c) the upper-half plane, and (d) a parallel plane capacitor.

The effective dielectric constant at the discharge gap is given as

$$\epsilon_{eff} = \frac{C_g}{C_{ga}} \quad (36)$$

where C_{ga} is the capacitance with the dielectric replaced by air.

III-4. Strip to ground capacitance

Figure 10 shows the schematic of the strip to the ground capacitance in two dimensions perpendicular to the stripline direction and the sequential mapping of the half of the strip to ground capacitance by Schwarz-Christoffel mapping method. Similarly, the mapping function of the rectangle in the z -plane to the upper-half of the t -plane is

$$t = \cos h \left(\frac{\pi(z - jh)}{2h} \right), \quad (37)$$

and the mapping function from the upper half of the t -plane to the parallel plane capacitor in the w -plane is

Figure 11. (Color online) Capacitance of one leg of a resonator versus a stripline having the same size. The black curve contains the results of the calculation in this work. The red curve contains the results of the stripline from an empirical model.

Figure 12. (Color online) Characteristic impedance of one leg of a resonator versus the stripline having the same size. The black curve contains the results of the calculation in this work. The red curve contains the results of the stripline from an empirical model.

$$w = F(\sqrt{-t}, k_5), \quad (38)$$

where $k_5 = \text{sech}\left(\frac{\pi w}{4h}\right)$ and $k_5' = \sqrt{1 - k_5^2}$. The capacitance per unit length of the stripline is

$$C_{su} = 2\epsilon_1\epsilon_0 \frac{K(k_5')}{K(k_5)}, \quad (39)$$

The capacitance between one leg of the resonator and ground, as shown in Fig. 11, is

$$C_s = 2\epsilon_1\epsilon_0 \frac{lK(k_5')}{K(k_5)}. \quad (40)$$

Figure 13. (Color online) The black curve shows the effect of the lengthening of the stripline on the resonant frequency and the red curve shows the effect of both the lengthening of the stripline and the discharge gap capacitance under changing of discharge gap size on the resonant frequency.

III-5. Characteristic impedance and resonance frequency

From the capacitance equivalence circuit for the resonator of MPSR operation in the odd-resonance mode shown in Fig. 3, the capacitance of one leg of the resonator can be given as

$$\begin{aligned} C &= C_s + 2C_m \\ &= 2\epsilon_1\epsilon_0 \frac{lK(k_5')}{K(k_5)} \\ &\quad + 2\epsilon_0\epsilon_1 \left(W \frac{(K(k_1') - v_3)}{2K(k_1)} + l \left[\frac{K(k_2')}{2K(k_2)} - \frac{W}{h} \right] \right). \end{aligned} \quad (41)$$

The characteristic impedance of each branch, as shown in Fig. 12, is

$$\begin{aligned} Z_s &= \frac{1}{cC} \\ &= \frac{60\pi}{\epsilon_1} \frac{1}{\epsilon_1 \frac{lK(k_5')}{K(k_5)} + \epsilon_1 \left(\frac{(K(k_1') - v_3)}{2K(k_1)} + l \left[\frac{K(k_2')}{2K(k_2)} - \frac{W}{h} \right] \right)}. \end{aligned} \quad (42)$$

The formation of the discharge gap shifts the resonant frequency of this device to a lower frequency. There are two main reasons for this effect. First is the lengthening of the two legs of the resonator owing to the contribution of two strip segments at the two ends of the resonator. The amount of shift in resonant frequency in this case is

$$\Delta f_1 = f_0 \left(1 - \frac{1}{1 + \frac{h-g}{l} \sqrt{\frac{\epsilon_{eff}}{\epsilon}}} \right) \quad (43)$$

where f_0 is the resonance frequency without a discharge gap, and ϵ_{eff} is the effective dielectric constant at the discharge gap:

$$\epsilon_{eff} = \frac{C_g}{C_{0g}} \quad (44)$$

where C_{0g} is the capacitance of discharge gap when dielectric is air.

The second reason is the shifting of resonance frequency due to the contribution of the capacitance at the discharge gap

$$\Delta f_2 = f_0 \left(1 - \frac{1}{\sqrt{1 + 2C_g/C}} \right). \quad (45)$$

Thus, the resonant frequency of MPSR can be given as

$$f' = f_0 - \Delta f_1 - \Delta f_2. \quad (46)$$

The result of the dependency of the resonant frequency on the gap width for a fixed thickness of the stripline (100 μm), calculated from Eq. (46), is presented in Fig. 13. The calculation and simulation results are in good agreement.

IV. Conclusions

In this paper, the capacitance and characteristic impedance of MPSR line were analyzed based on the conformal mapping method, and the evolution trend

of the resonance frequency was investigated to understand the discharge matching condition. The calculation results are in good agreement with the results of the commercial CST microwave studio software. This paper contributes toward the understanding of the operation principle and optimization of MPSR.

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